SEMESTER	IV	QP CODE	4225

P.R. GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA

SEM IV EXAMINATIONS MAY -2024

II B.SC. MATHS SUBJECT: LINEAR ALGEBRA

DATE & SESSION	30.05.2024 FN	REG NO								MAX MARKS	50

Time: 2Hrs **Max. Marks: 50**

SECTION-A

Answer any three questions selecting atleast one question from each part

Part – A **3 X 10 = 30M**

1. Prove that the necessary and sufficient condition for a non-empty subset W of a vector space V(F) to be a subspace of V is that $a, b \in F, \alpha, \beta \in W \Rightarrow a\alpha + b\beta \in W$.
2. Let W be a subspace of a finite dimensional vector space V(F) then prove that $\dim(V/W) = \dim V - \dim W$.
3. Let $T: R^3 \rightarrow R^3$ be defined by $T(x, y, z) = (x - y + 2z, 2x + y - z, -x - 2y)$. Then verify Rank-nullity theorem.

Part – B

4. Find the eigen roots and the corresponding eigen vectors of the square matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

5. State and prove Cayley-Hamilton theorem.
6. Apply the Gram-Schmidt process to the vectors $\beta_1 = (2, 1, 3)$, $\beta_2 = (1, 2, 3)$, $\beta_3 = (1, 1, 1)$ to obtain an orthonormal basis for $V_3(R)$ with the standard product

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SECTION-B

Answer any four questions. 4 X 5 = 20M

1. Show that the vector $\alpha = (2, -5, 3)$ in R^3 cannot be expressed as a linear combination of the vectors $e_1 = (1, -3, 2)$, $e_2 = (2, -4, -1)$ and $e_3 = (1, -5, 7)$
2. Prove that the linear span $L(S)$ of any subset S of a vector space $V(F)$ is a subspace of $V(F)$
3. State and Prove invariance theorem.

4. Show that the vectors (1,1,2), (1,2,5), (5,3,4) of $R^3(R)$ do not form a basis of $R^3(R)$.
5. Let $U(F)$ and $V(F)$ be two vector spaces such that T: $U(F) \rightarrow V(F)$ be a linear transformation. Prove that the range set $R(T)$ is a subspace of $V(F)$.
6. Show that the system of equations $x - 4y + 7z = 14$, $3x + 8y - 2z = 13$, $7x - 8y + 26z = 5$ are inconsistent.
7. State and prove Parallelogram law in an inner product space V(F).